Analyse

Exam

29th of June of 2006

1 Let $\{x_n\}_{n\in\mathbb{N}}$ be a convergence sequence of $\mathbb{R}^n$ and $x = \lim_{n\to\infty} x_n$. Prove that $A = \{x_n\}_{n\in\mathbb{N}} \cup \{x\}$ is a compact set. (2 points)

2 Consider $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by $f(x,y) = \frac{yx^2}{x^2+y^4}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

(i) Prove that f is continuous at $(0,0)$.

(ii) Compute $D_u f(0,0)$ for all $u \in \mathbb{R}^2$.

(iii) Is f differentiable at $(0,0)$?. (give an appropriate argument.)

(3 points)

3 (i) Prove that for all $n \in \mathbb{N}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(ii) Prove in two different ways that the function y^2x^2 is integrable in $[0,1] \times [0,1]$.

(3 points)

4 Write down the inverse function theorem. (A proof of it is not required.) (1 point)